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THE STRUCTURE OF RADICAL PROBABILISM*

ABSTRACT. Does the philosophy of Radical Probabilism have enough structure to enable it to address fundamental epistemological questions? The requirement of dynamic coherence provides the structure for radical probabilist epistemology. This structure is sufficient to establish (i) the value of knowledge and (ii) long run convergence of degrees of belief.

1. INTRODUCTION

Richard Jeffrey advocates a skeptical epistemology grounded in *radical probabilism*. The fundamental concept of epistemology is not to be taken as knowledge, but rather degree of belief. It is rarely plausible that degrees of belief should take the extreme form of certainty. In particular, learning does not proceed by conditioning on observation statements which are learned with certainty. All sorts of learning processes are deemed possible, some – but not all – falling under Jeffrey's well-known model of probability kinematics.

In avoiding oversimplifications and illicit assumptions, radical probabilism meets high epistemological standards. But does this degree of realism leave us with any interesting structure in the general framework? In this essay I will review results about how dynamic coherence provides structure in the radical probabilist picture, and how some central features of a conditioning model carry over to the more general approach of radical probabilism.

2. BASIC DYNAMIC COHERENCE RESULTS

The foundation for dynamic coherence arguments is a well-known argument by de Finetti (1937) for the definition of conditional probability as $\Pr(q|p) = \Pr(p \,\&\, q)/\Pr(p)$ when $\Pr(p) > 0$. Conditional probabilities are used to evaluate conditional bets. But de Finetti pointed out that one can achieve the effect of a bet on q conditional on p by making two unconditional bets, one on $p \,\&\, q$ and another against p, at stakes such that the net payoff is zero if the condition, p, is not realized. For the two routes

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of evaluation to agree the usual definition of conditional probability is required.

Ian Hacking (1967) argued that this result is totally static, that it deals only with the coherence of conditional and unconditional probabilities at a single time, and that it gives no support whatsoever for Bayes' rule of updating by conditioning on the evidence. It takes only a small twist, however, to turn de Finetti's observation into a dynamic argument for Bayes' rule. Among philosophers, this step was taken by David Lewis and communicated by Paul Teller (1973). In the statistical literature the argument is often taken to be implicit in de Finetti, although what de Finetti actually says does not make exegesis straightforward.[1]

Suppose that there is a finite set of evidence statements, each with positive prior probability, one of which is to be learned for certain. And suppose that the epistemic agent is considering potential rules for updating subjective probability on the basis of the evidence learned. Mathematically, such a rule is a function from the possible evidence $E = \{e_1, e_2, \ldots, e_n\}$ to revised probability measures. Such a rule is incoherent if a bettor knowing the rule and making a finite number of bets initially and a finite number of bets after the evidence is in, can achieve a sure net gain. Mathematically, a bettors strategy is a pair of functions, the first mapping the agent's initial probability and rule onto a finite set of initial bets; the second mapping the initial probabilities, rule and evidence learned onto a finite set of bets. The result is that it is necessary and sufficient for coherence that the agent adopt Bayes' rule of updating by conditioning on the evidence. The leading idea of the proof is that in this situation the bettor can make a bet on p conditional on e in one of two ways. The first is to make to conditional bet in the de Finetti way; the second is to adopt a strategy of waiting until the evidence is in and betting on p just in case the evidence is e. If these two ways disagree it is obvious that the way is open to a strategy which guarantees a sure win conditional on e, and since e has positive initial probability a suitable initial sidebet against e converts this to a strategy which unconditionally guarantees a sure win. One might have some reservations as to the applicability of the argument on account of the restrictions that the set of potential evidential statements be (i) finite and (ii) such that each has positive prior probability [Kyburg (1978)], but it turns out that these conditions are inessential. Lane and Sudderth (1985) show that the result holds quite generally.

What happens when we pass from the foregoing to the radical probabilist model? Here the epistemic agent starts with an initial probability, pr_1, passes through a "black-box" learning situation, and comes out with a final probability, pr_2. We are not supposed to speculate on what goes on inside the black box. Nevertheless, there is a dynamic coherence result due to Goldstein (1983) and van Fraassen (1984) parallel to that for conditioning. Suppose that the agent's prior probability for his posterior probability of p is concentrated on a finite number of values, $a_1 \ldots a_m$. Then coherence requires that:

$$\text{(M)} \qquad \mathrm{pr}_1(p|\mathrm{pr}_2(p) = a_i) = a_i \quad (\text{for } i = 1 \text{ to } m)$$

which has as a consequence that the prior probability is the expectation of posterior probability.[2]

The bets used to make the dutch book are the same as before, except that instead of bets conditional on a statements of evidence, e, we have bets conditional on a statement of final probability, $\mathrm{pr}_2(p) = a$. Dynamic coherence forces the black box learner to behave *as if* she were conditioning on the statement of final probability, as in Skyrms (1980).[3]

There has been some question as to whether the foregoing dynamic coherence arguments hold up in the context of game theory or sequential decision theory, the thought being that if an incoherent agent "sees a dutch book coming" she will simply refuse to bet at all and thus avoid the sure loss. See Maher (1992), Earman (1992). Analysis of the argument, however, shows that such is not the case. [For details see Skyrms (1993). That discussion is framed in terms of the Lewis conditioning model, but the same analysis works for the radical probabilist black box model.] The incoherent agent, subsequent to the black box experience, will accept the cunning bettors offer as a way of cutting her losses, while regretting the initial bets she made prior to going into the black box. But initially, even knowing the bettor's strategy, she will accept his initial offers as a means of cutting her losses while rueing the decisions that she believe she will be disposed to make once she has gone through the black box. The analysis also has consequences for the discussion of the next section.

Between the transparency of the conditioning model of learning and the opacity of the "black box", we have models of various degrees of translucency generated by *Jeffrey's rule* of updating by probability kinematics on a partition. Jeffrey's basic model assumes a finite partition each of whose members has positive prior probability. A probability, pr_2 is said to come from another pr_1 by probability kinematics on this partition just in case the final probabilities conditional on members of the partition, where defined, remain the same as the initial probabilities conditional on members of the partition. Conditioning on a member of the partition is the special case of probability kinematics in which that member gets final probability of one. Jeffrey had in mind a model in which one could approximate certain evidence without being forced to regard learning as learning for certain.

More general forms of the rule are possible. To say that pr_2 comes from pr_1 by probability kinematics on the partition is to say that it is a *sufficient partition* for $\{pr_1, pr_2\}$. The natural generalization says that pr_2 comes from pr_1 by probability kinematics on a sub-sigma-algebra, if it is a *sufficient sub-sigma-algebra* for $\{pr_1, pr_2\}$ [Diaconis and Zabell (1982)]. Here, however, we focus on the simplest case.

From the point of view of conditioning, Jeffrey's rule relaxes structure; from the point of view of the black box model, Jeffrey's rule (with respect to some fixed partition) imposes structure. In what sense can dynamic coherence be brought to bear on probability kinematics?

Suppose that the agent about to go into the black box, believes that the only information she will gain will be information about a partition of colors, although the information may not be certain. One way to express this is to introduce a later "reference point" in which she find out the true member of the color partition. If the black box only provided information about color, the going through the black box and then finding out the true color with certainty should result in the same probability, pr_3, as one would have gotten by bypassing the black box and going directly from pr_1 *via* certain learning to pr_3. Then by the Lewis-Lane-Sudderth argument, the probabilities conditional on members of the partition should be the same in pr_3 as in pr_2 and they also should be the same in pr_3 as in pr_1. Therefore pr_2 must come from pr_1 by probability kinematics on the partition.

This is the leading idea of a dutch book argument for probability kinematics in Skyrms (1987) and for a somewhat different dutch book theorem for probability kinematics based on ideas of Armendt (1980) in Skyrms (1990). What these theorems show is that if the agent believes with probability one that the learning experience *only* gives information about the partition in question, then coherence requires that belief change proceed by probability kinematics on that partition.

3. THE VALUE OF KNOWLEDGE

The fundamental theorem of epistemology is that knowledge is good for you. That is to say that the expected utility of acquiring pure cost free information[4] is non-negative, and indeed positive if there is any positive probability that the information will change your mind as to the optimal act to perform. The theorem is proved in the context of the classical conditioning model by Savage (1954) and Good (1967).

It is, in fact, anticipated in a manuscript of Frank Ramsey that I discovered in the Ramsey archives at the University of Pittsburgh. The note is on two pages which were separated by another on a quite different topic.

There is some indication that Ramsey was interested in extending the theorem to something like Jeffrey's rule, but this is a matter of interpretation. It is discussed in Skyrms (1990) pp. 93–96. These notes of Ramsey were subsequently transcribed and published by Nils-Eric Sahlin and by Maria Carla Galavotti.

In 1989 Paul Graves showed how the value of knowledge theorem can be demonstrated in a model in which agents update by Jeffrey's rule. In this model agents satisfy condition (M) of the previous section as well as a sufficiency condition for the partition used by Jeffrey's rule. Subsequently it became clear to Graves and to myself that condition (M) alone is all that is required for the value of knowledge theorem [Skyrms (1990) Ch. 4]. The heart of the argument is very simple. Let $B(\mathrm{pr})$ be the expected utility of the Bayes act – the act that maximizes expected utility – according to the probability pr. Then, under the assumptions of the theorem which I discuss in the foregoing reference:

$$U(\text{Act now}) = B[E(\mathrm{pr}_f)]$$

and

$$U(\text{Learn now, act later}) = E[B(\mathrm{pr}_f)]$$

That the utility of learning now and acting later is greater than or equal to the utility of acting now is an immediate consequence of the convexity of B by Jensen's inequality.

In this setting, condition (M) is sufficient for the value of knowledge theorem. It is necessary? In other words, if condition (M) fails in the black box situation can we find some decision problem such that with respect to it the expected utility of the expected utility of acting is greater than the expected utility of going through the black box and then acting? An affirmative answer follows immediately from the previous discussion of dynamic coherence. Suppose that the agent's beliefs about an impending black box violate condition (M) in the simplest case where the agents prior probabilities are concentrated on a finite number of possible final probabilities. For example, suppose that $\mathrm{pr}_i(Q|\mathrm{pr}_f(Q) = 2/3) = 1/3$ and $\mathrm{pr}_i(\mathrm{pr}_f(Q) = 2/3) > 0$. The violation of condition (M) gives us conditional bets which look unattractive *ex ante* but which the agent believe will look attractive *ex post* if the condition is realized. For example consider an even money bet on Q conditional on $\mathrm{pr}_f(Q) = 2/3$. Now suppose that the decision problem is whether to accept or reject this bet. The decision maker will assign high expected utility to act now (and reject the bet) rather than going through the black box and acting later (and risking acceptance of the bet). I suggest elsewhere, that failure of condition (M) be interpreted as reflecting the agent's belief that this black box is not properly thought of as a "learning situation" but rather as some other kind of belief change.

Returning to the theme of this paper, in the radical probabilist framework the fundamental theorem of epistemology holds just when we have dynamic coherence.

4. CONVERGENCE

But can radical probabilists prove anything about convergence in the long run? In *Bayes or Bust* John Earman is skeptical about the resources of skeptical philosophy:

> ... a Bayesianism that appeals to both Dutch Book and strict conditionalization is on a collision course with itself. The use of strict conditionalization leads to situations where $\Pr(A) = 1$ although $\not\models A$. As a result, something almost as bad as a Dutch book befalls the conditionalizer; namely she is committed to betting on the contingent proposition A at maximal odds, which means that in no possible outcome can she have a positive gain and in some possible outcome she has a loss (a violation of what is called *strict coherence*). It is too facile to say in response that this is a good reason for abandoning strict conditionalization in favor of Jeffrey conditionalization or some other rule for belief change; for all the results about merger of opinion and convergence to certainty so highly touted in the Bayesian literature depend on strict conditionalization ... (Earman 1992, 41).

There is, however, a general convergence theorem for radical probabilist learning with connections to a fuller treatment of dynamic coherence.

Contemplate, at the onset, the prospect of an infinite sequence of black box learning situations. In each episode you go into the black box with a probability of proposition A and come out with a revised probability of proposition A. Here we make no assumptions about what goes on in the black box. We do not assume that you conditionalize on some evidential proposition delivered to you in the box. We do not assume anything else about the structure of your learning experience either. Now we can look for conditions which will get almost sure convergence. Let us look for a martingale.

Consider a probability space -- here your degree-of-belief space, and let $x_1, x_2, \dots$ be a sequence of random variables on that space and $F_1, F_2, \dots$ be a sequence of subsigma fields. The sequence of random variables is a *martingale relative to the sequence of sigma-fields* if:

(i) The sequence of sigma-fields is non-decreasing
(ii) x_n is measurable F_n
(iii) $E[|x_n|]$ is finite

(iv) with probability 1: $E[x_{n+1}||F_n] = x_n$

The sequence of random variables is a *martingale* if it is a martingale relative to some sequence of sigma fields.

You are interested in whether you can have confidence that your sequence of revised probabilities will converge, so let us take the random variable x_n to be the revised probability of proposition A after coming out of the nth black box. Since this is a probability, condition (iii) is automatically satisfied. We do not have any evidence statements given in our model to generate sigma-fields, so we might as well consider the sigma-fields generated by our random variables: $F_n = \sigma[x_1, \ldots, x_n]$. With these sigma fields, (i) and (ii) are automatically satisfied and we are left as the requirement for a martingale:

(iv′) $E[x_{n+1}||x_1, \ldots, x_n] = x_n$

If (iv′) is not satisfied, you may very well think that your beliefs are likely to oscillate forever – for instance with revised probability of A being high after even black boxes and low after odd black boxes. But if (iv′) is satisfied and if your degrees of belief are countably additive[5], then by the martingale convergence theorem you believe with probability one that your sequence of revised probabilities of A will converge. Condition (iv′) is a sufficient condition for almost sure convergence of opinion in a black-box learning situation, but does it have any special status for a radical probabilist?

5. COHERENCE REVISITED

In this section we see the martingale condition (iv′) is a necessary condition for dynamic coherence of degrees of belief in a setting where we have an infinite sequence of black box learning situations. We will assume sigma-coherence here, in order to ensure sigma additivity. That is to say a bettor can make a countable number of bets in his attempt to dutch book you, and you are sigma coherent if no dutch book can be made.

As a preliminary, consider the case of two black boxes. You now contemplate going through 2 black box learning situations, coming out at time t_1 with a revised probability of A, x_1, and coming out at time t_2 with a further revised probability of A, x_2. Also at t_1 you will have a revised expectation of x_2, which we will call y_1. We assume that y_1 is measurable with respect to the sigma-field generated by x_1 and integrable. From your current standpoint at t_0, y_1 is also a random variable.

(Ca) Coherence requires that y_1 is a version of the conditional expectation:

$$E[x_2||x_1].$$

Let G be a set in the σ-field generated by x_1. At t_1, a contract which pays off x_2 at t_2 has a fair price of y_1 to the agent. At t_0, a contract (CON1) with a fiducial agent to buy or sell such a contract at t_1 at its t_1 fair price, conditional on G being the case at t_1, has a fair price of:

$$\int_G y_1 dp \qquad \text{(CON1)}$$

At t_0, a contract, (CON2), conditional on G which pays off x_2 at t_2, has a fair price of:

$$\int_G x_2 dp \qquad \text{(CON2)}$$

Since these contracts have the same consequences, coherence requires that they have equal value.

(Cb) Coherence requires that $y_1 = x_1$ almost everywhere.

If the agent were always coherent at t_1, then $y_1 = x_1$ by the Goldstein-van Fraassen argument. If the agent is incoherent at t_1 for a set, S, of positive measure in p, then the agent can be dutch-booked at t_0: bet at t_0 against S; if S is not true at t_1 collect; if S is true at t_1 pay off the original bet and proceed with the dutch book at stakes large enough to assure a net profit.

(C) Coherence requires that (for some version) $E[x_2||x_1] = x_1$.

From (Ca) and (Cb), x_1 is a version of $E[x_2||x_1]$.

The foregoing reasoning generalizes. You now contemplate an infinite sequence of black box learning experiences together with the associated sequences of revised probabilities of $A, x_1, x_2, x_3, \ldots$ Then the coherence argument for conditional expectation [as under (Ca)] gets us:

(CCa) Coherence requires that y_{n+1} is a version of the conditional expectation:

$$E[x_{n+2}||x_1, \ldots, x_{n+1}].$$

and the coherence argument for future coherence [as under (Cb)] gets us:

(CCb) Coherence requires that $y_{n+1} = x_{n+1}$ almost everywhere.

Putting these together we have:

(CC) Coherence requires the martingale condition, (iv′).

6. ANOTHER MARTINGALE?

Let I_A be the indicator function for $A, F_n = \sigma[x_1, \ldots, x_n]$ as before and F_∞ be the sigma field generated by the union of the F_ns. The random variables $E[I_A||F_n]$ form a martingale relative to the sigma fields F_n. Because of the uniform integrability properties of conditional expectations we can not only say that this martingale converges with probability one, but we can also say something about the random variable to which it converges:

$$E[I_A||F_n] \to E(I_A||F_\infty) \text{ (with probability} = 1)$$

We might gloss this by saying that with this martingale we have convergence to a maximally informed opinion.

Furthermore, we can say this without invoking any dynamic coherence arguments (although we presuppose static sigma-coherence). The reason is that our conclusion does not say anything about the temporal process of belief change, since there is nothing to link the conditional expectations, $E[I_A||F_n]$, to subsequent belief states.

Suppose, however, that we now assume dynamic coherence. Let $E_n(I_A)$ be the expectation of the indicator, I_A, that you have at t_n according to your probabilities at t_n. By a coherence argument for conditional expectation like that given in Section 3:

(CCC) $E_n(I_A) = E[I_A||F_n]$

and, by definition:

$$x_n = E_n(I_A).$$

Under the assumption of dynamic coherence, the martingale of this section is the same martingale as that of Section 4:

$$\langle x_n, F_n\rangle = \langle E[I_A||F_n], F_n\rangle.$$

So we have:

$$x_n \to E(I_A||F_\infty) = p(A||F_\infty) \text{ (with probability 1).}$$

7. CONVERGENCE AND KINEMATICS

What is the relation of probability kinematics to the martingale property? First, let us notice that the convergence results which we discussed for a single proposition, A, apply more widely. Consider a finite number of propositions, $A_1, \ldots, A_n$. Their probabilities are given by a vector, $\mathbf{x}$, in $[0,1]^n$. The foregoing martingale convergence story continues to hold for the vector valued random variables, $\mathbf{x}_1, \mathbf{x}_2, \ldots$ [see Neveu (1975) for vector valued martingales].

Probability kinematics can be thought of as a technique for making the black box translucent. For example, suppose the black box learning situations consist of repeatedly looking at a jellybean by candlelight. R is the proposition that it is Red; C is the proposition that it is cinnamon flavored. $\mathbf{x}_1$, $\mathbf{x}_2$ ... are the probability vectors for these propositions at subsequent times, with the first coordinate being color and the second flavor: e.g. $\mathbf{x}_2$ [1] is the probability at time 2 the it is Red.

Suppose that you are certain that belief change will be by probability kinematics on $\{R, -R\}$; that probabilities conditional on R and on $-R$ will remain unchanged. You do not automatically satisfy the martingale condition. You might believe that your probability for R will be .99 at even numbered times and .01 at odd numbered times. In such a case you would expect your beliefs to oscillate forever, and you would be susceptible to a dynamic dutch book.

But if your beliefs do have the martingale property as well, then with probability one the vector valued martingale, $\mathbf{x}_1$, $\mathbf{x}_2$, ... converges to a vector valued random variable $\mathbf{x}_\infty$. With probability one, the random variable $\mathbf{x}_\infty$ must take values which preserve the original probabilities of flavor conditional on R and $-R$; that is to say the limiting beliefs come from the initial ones by probability kinematics on this partition.

If we consider sequences of belief change by probability kinematics where the kinematics does not take place with respect to a single fixed partition the situation is much more complex. Some relevant results can be found in Rota (1962) and in Diaconis and Zabell (1982).

8. CONCLUSION

Radical Probabilism takes its structure from considerations of dynamic coherence. Where applicable, belief change by probability kinematics on a partition or a sigma field adds more structure. But the structure imposed by coherence alone is sufficient for two very general theorems that are hallmarks of the Bayesian point of view: the convergence theorem and the theorem on the value of knowledge.

NOTES

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[1] For example see de Finetti (1975) pp. 202–204.

[2] As before, the basic argument carries over to more general settings. See Skyrms (1980) Appendix 2, Goldstein (1983), Gaifman (1988), Skyrms (1990) Ch. 5.

[3] In this regard, it may be of interest to juxtapose the coherence argument of Lewis for conditioning with the second order coherence argument of Uchii (1973). I take this to be the point of some of Colin Howson's remarks at this conference.

[4] We assume the act of acquiring the information – performing the experiment or making the observation – does not itself affect the probabilities or values of outcomes of the decision in question. For further discussion see Maher (1990) and Skyrms (1990).

[5] I will not address here the question of countable additivity in radical probabilism, but I would like to point out that the Bolker representation for Jeffrey's system of personal probability yields countable additivity.

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